

Determination and application of TMDs obtained by the Parton Branching method

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in collaboration with

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Outline

- Parton Branching (PB) method
- TMD determination and validation
- TMD fit to data
- Application to LHC processes
- Summary and conclusions

TMDs

- small momentum transfer
- small- x
- high energy limit

Parton Branching method

- Novel method to solve the TMD evolution equation
- fully exclusive solution
- valid at LO, NLO and NNLO

Parton Branching method and TMDs

Parton Branching method

- PB evolution equation:

$$Q^2 \frac{\partial \hat{f}_a(x, Q^2)}{\partial Q^2} = \sum_b \int_x^{z_{\max}} dz P_{ab}^R(z, \alpha_s(Q_r^2)) \hat{f}_b\left(\frac{x}{z}, Q^2\right) - \hat{f}_a(x, Q^2) \sum_b \int_0^{z_{\max}} dz z P_{ba}^R(z, \alpha_s(Q_r^2))$$

- z_{\max} separates the **resolvable** and **non-resolvable** phase space regions
- non-resolvable solution \Rightarrow Sudakov factor:

$$\Delta_a(Q^2, Q_0^2) \equiv \exp \left[- \sum_b \int_{Q_0^2}^{Q^2} \frac{dQ'^2}{Q'^2} \int_0^{z_{\max}} dz z P_{ba}^R(z, \alpha_s(Q_r'^2)) \right]$$

Parton Branching method

- evolution equation, integral form:

$$\hat{f}_a(x, Q^2) = \hat{f}_a(x, Q_0^2) \Delta_a(Q^2, Q_0^2) + \int_{Q_0^2}^{Q^2} \frac{dQ'^2}{Q'^2} \frac{\Delta_a(Q^2, Q_0^2)}{\Delta_a(Q'^2, Q_0^2)} \int_x^{z_{\max}} dz \sum_b P_{ba}^R(z, \alpha_s(Q'^2)) \hat{f}_b\left(\frac{x}{z}, Q'^2\right)$$

- iterative solution:

$$\hat{f}_a^{(0)}(x, Q^2) = \hat{f}_a(x, Q_0^2) \Delta_a(Q^2, Q_0^2)$$

$$\hat{f}_a^{(1)}(x, Q^2) = \hat{f}_a(x, Q_0^2) \Delta_a(Q^2, Q_0^2) + \int_{Q_0^2}^{Q^2} \frac{dQ'^2}{Q'^2} \frac{\Delta_a(Q^2, Q_0^2)}{\Delta_a(Q'^2, Q_0^2)} \int_x^{z_{\max}} dz \sum_b P_{ba}^R(z, \alpha_s) \hat{f}_b\left(\frac{x}{z}, Q_0^2\right) \Delta_b(Q'^2, Q_0^2)$$

$$\hat{f}_a^{(2)}(x, Q^2) = \dots$$

⋮

Parton Branching method

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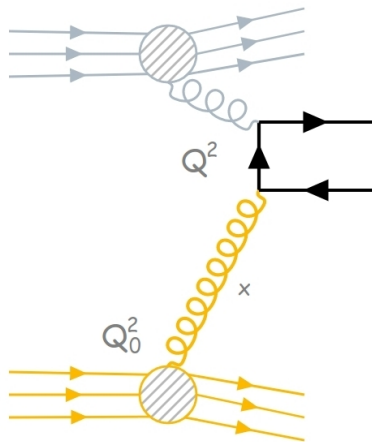
⋮

o example: first iteration, $a, b = g$

$$\hat{f}_g^{(1)}(x, Q^2) = \hat{f}_g(x, Q_0^2) \Delta_g(Q^2, Q_0^2) + \\ + \int_{Q_0^2}^{Q^2} \frac{dQ'^2}{Q'^2} \frac{\Delta_g(Q^2, Q_0^2)}{\Delta_g(Q'^2, Q_0^2)} \int_x^{z_{\max}} dz P_{gg}^R(z, \alpha_s) \hat{f}_g\left(\frac{x}{z}, Q_0^2\right) \Delta_g(Q'^2, Q_0^2)$$

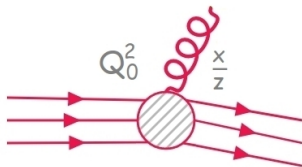
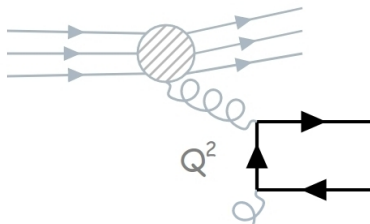
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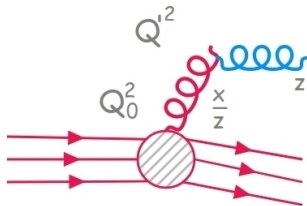
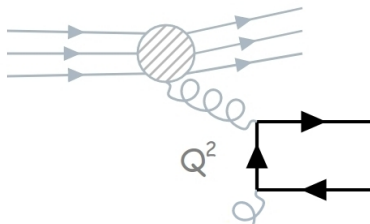
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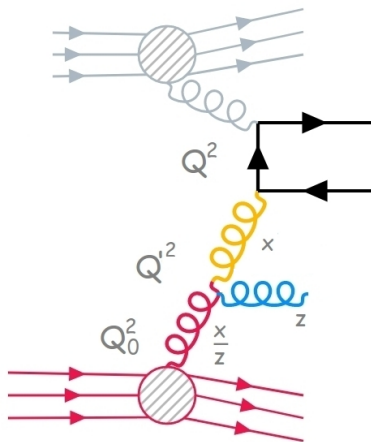
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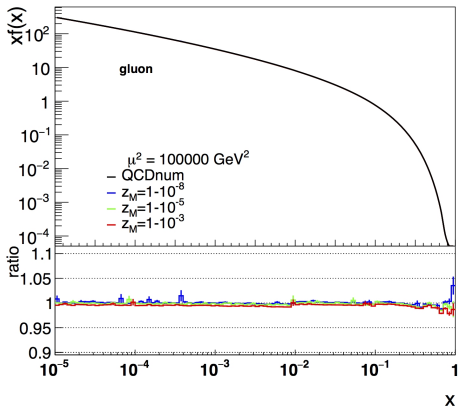
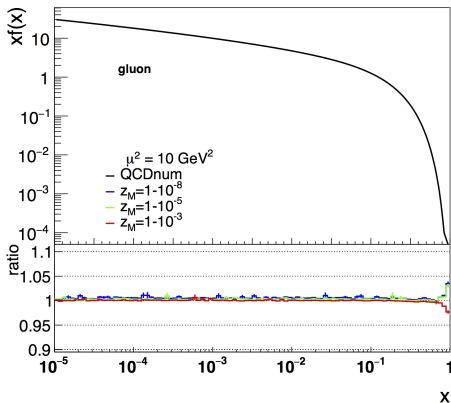
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PB iterative solution:

- kinematics of the splittings is known
- cumulative k_T of the branchings \Rightarrow TMD
- physics \rightarrow evolution variables to splitting kinematics mapping

Validation with QCDnum at NLO



- correction $\mathcal{O}(1 - z_{\max})$
- z_{\max} large \Rightarrow good agreement!

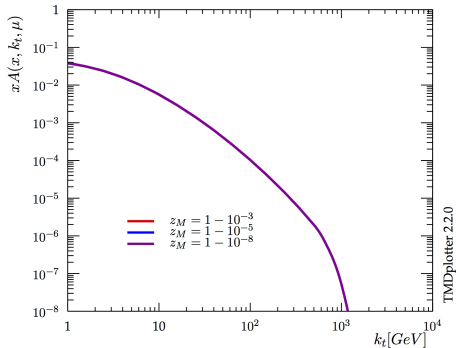
Mapping $z, p_T \rightarrow Q, Q_r$

○ evolution variable Q :

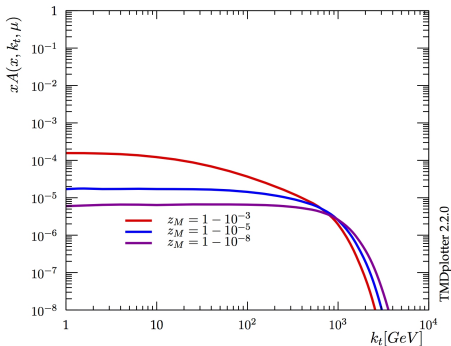
- p_T ordered $Q = p_T$

- angular ordered $Q = p_T/(1-z) \Rightarrow$ coherence

gluon, $x = 0.01, \mu = 1000 \text{ GeV}$



gluon, $x = 0.01, \mu = 1000 \text{ GeV}$

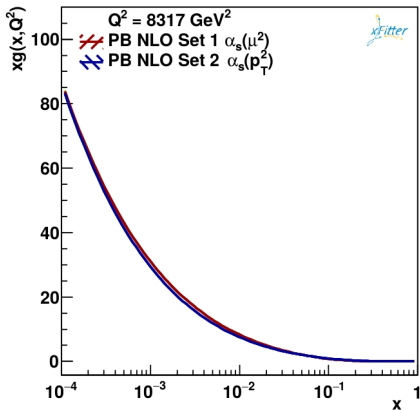
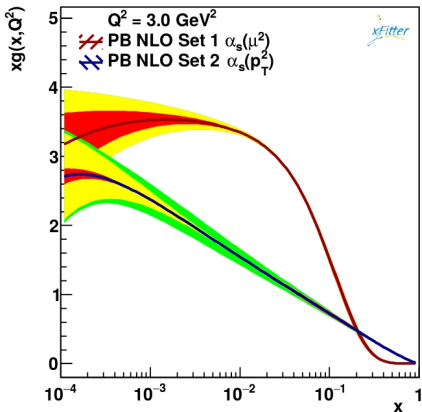


Mapping $z, p_T \rightarrow Q, Q_r$

- renormalization scale at the splitting vertex Q_r :

$$Q_r = Q$$

$$Q_r = p_T \text{ (under the aforementioned coherence assumption)}$$



- partonic content differs at small scales
- uncertainty sources \rightarrow next slides

Fit to data

Fit to data

- DIS measurements from HERA I+II
- kinematic range:
 $3.5 < Q^2 < 50000\text{GeV}^2$, $4 \cdot 10^5 < x < 0.65$
- fitting procedure in a nutshell:

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(implemented in xFitter)

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 - store the TMD in a grid for later use
(TMDlib, complementary slides)

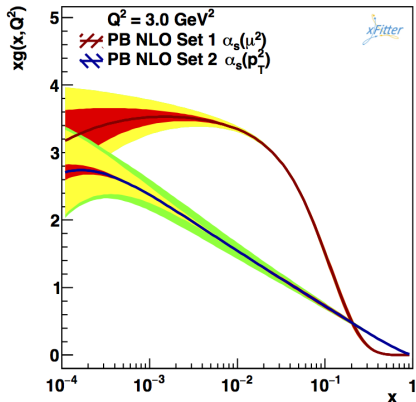
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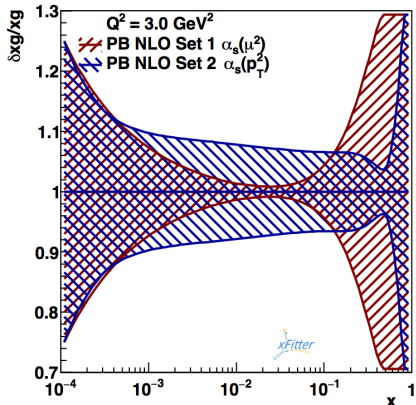
⇒ First TMD fit to precision data!

Parton density uncertainty sources

- **experimental uncertainty** → Hessian method $\Delta\chi^2 = 1$
- **model dependence** → b, c masses at Q_0
+ Q_r threshold for set 2



total uncertainty

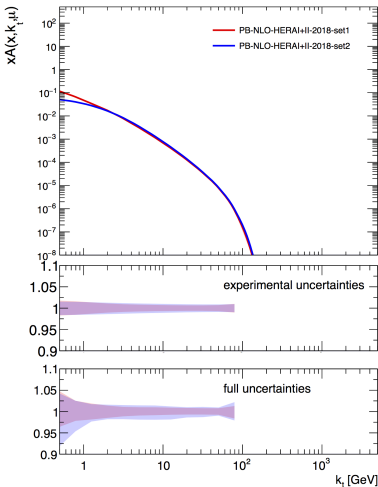


TMD parton distributions

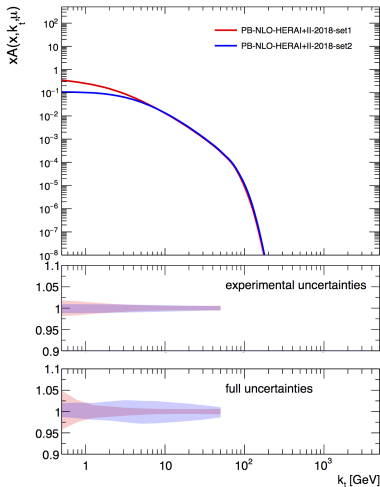
- set 1 $Q = p_T/(1-z)$, $Q_r = Q$ ($\chi^2/ndf = 1.2$)

- set 2 $Q = p_T/(1-z)$, $Q_r = p_T$ ($\chi^2/ndf = 1.21$)

anti-up, $x = 0.01$, $\mu = 100$ GeV



gluon, $x = 0.01$, $\mu = 100$ GeV

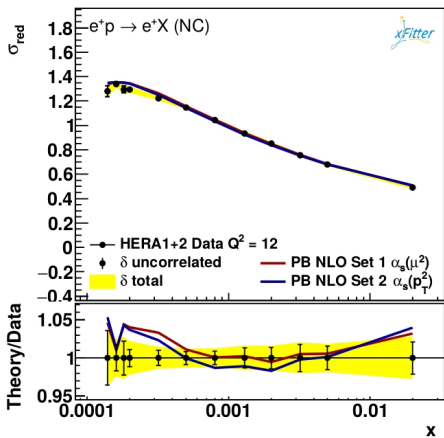


- model dependence dominant (set 2)

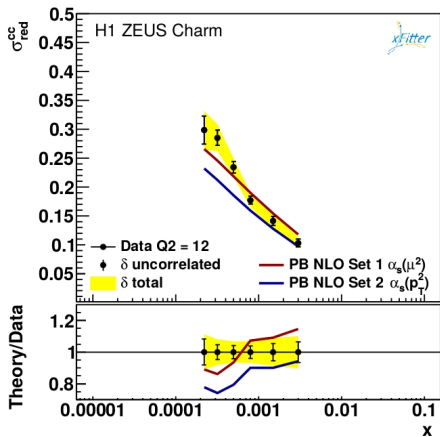
Comparison with HERA data

- set 1 $Q = p_T/(1-z)$, $Q_r = Q$ ($\chi^2/ndf = 1.2$)
- set 2 $Q = p_T/(1-z)$, $Q_r = p_T$ ($\chi^2/ndf = 1.21$)

inclusive DIS



inclusive charm



- inclusive DIS well described
- set 2 disagrees at small x for inclusive charm

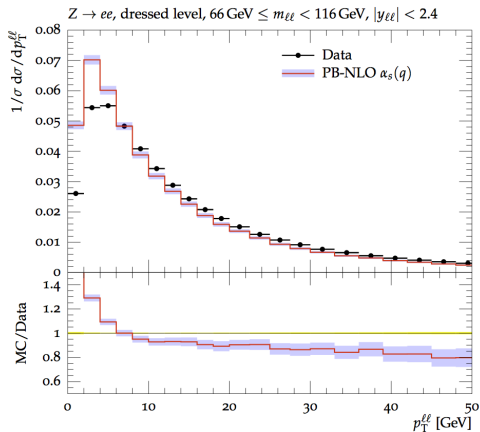
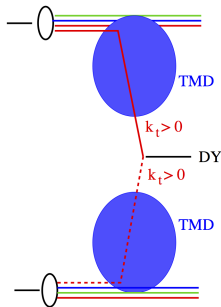
Details

- F. Hautmann, H. Jung, A. Lelek, V. Radescu, and R. Zlebcik. Soft-gluon resolution scale in QCD evolution equations. Phys. Lett., B772:446451, 2017
- F. Hautmann, H. Jung, A. Lelek, V. Radescu, and R. Zlebcik. Collinear and TMD Quark and Gluon Densities from Parton Branching Solution of QCD Evolution Equations. JHEP, 01:070, 2018.
- A. Bermudez-Martinez, P. Connor, F. Hautmann, H. Jung, A. Lelek, V. Radescu, and R. Zlebcik. Collinear and TMD parton densities determined from fits to HERA DIS measurements, DESY-18-042

Application: low momentum transfer

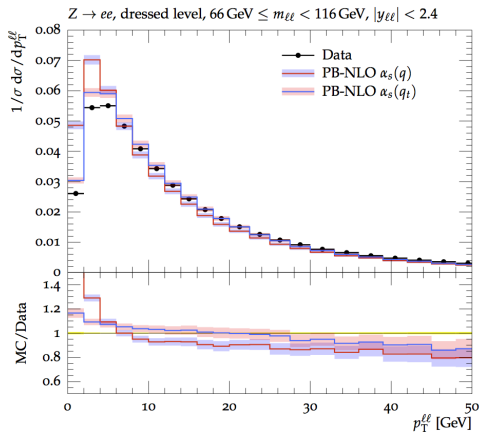
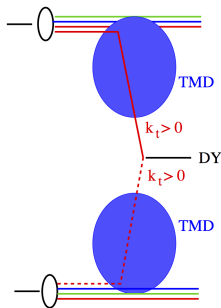
Drell-Yan p_T

- $Q = p_T/(1 - z)$, $Q_r = Q$



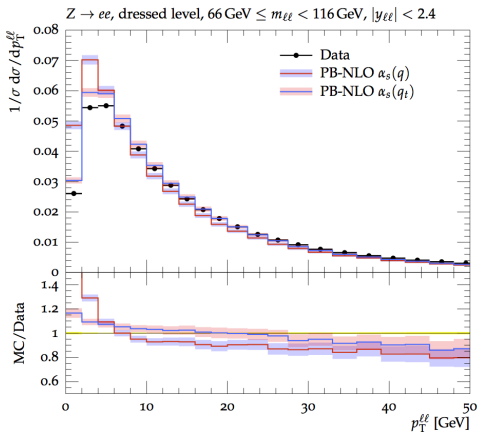
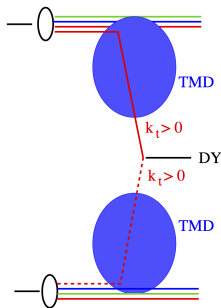
Drell-Yan p_T

- $Q = p_T/(1-z)$, $Q_r = Q$
- $Q = p_T/(1-z)$, $Q_r = p_T$ (under angular ordering)



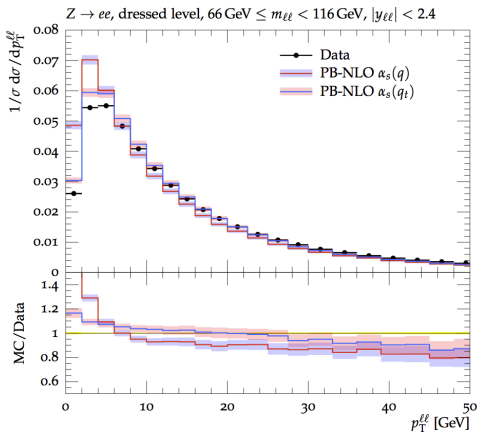
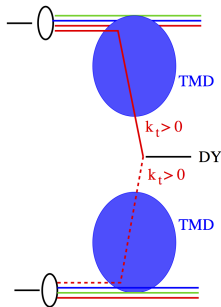
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- shape described by both variants



Drell-Yan p_T

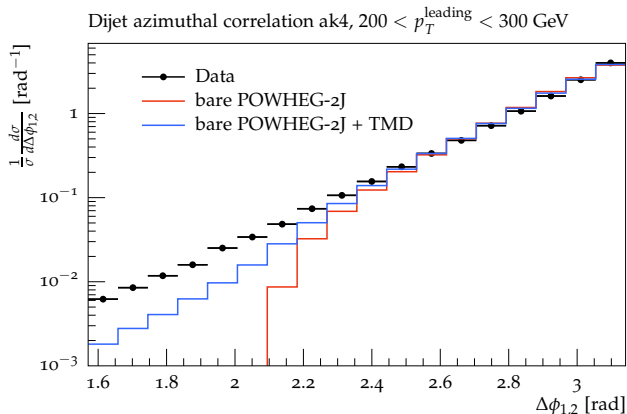
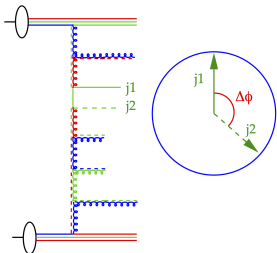
- $Q = p_T/(1-z)$, $Q_r = Q$
- $Q = p_T/(1-z)$, $Q_r = p_T$ (under angular ordering)
- shape described by both variants
- low $p_T^{\ell\ell}$ better described by $Q_r = p_T$



Application: high momentum transfer

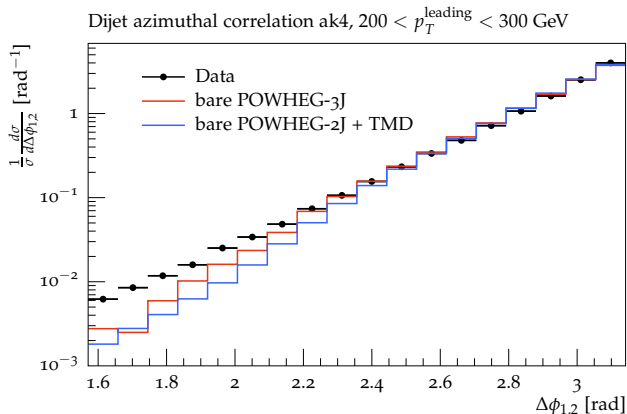
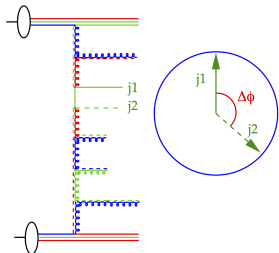
Dijets $\Delta\phi_{1,2}$

- opens up the ME phase space



Dijets $\Delta\phi_{1,2}$

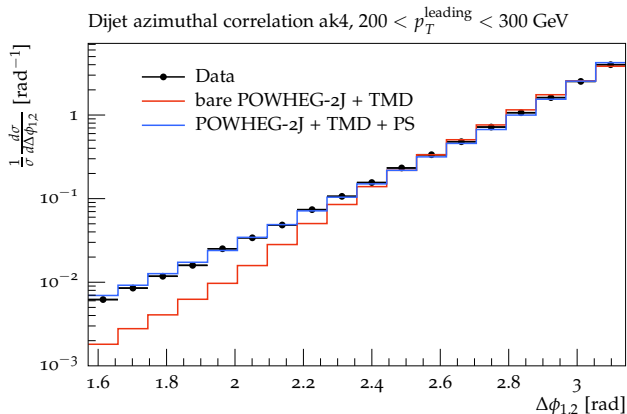
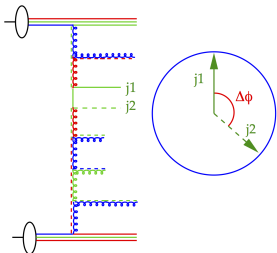
- opens up the ME phase space
- catches higher-order contributions



CMS Collaboration Eur. Phys. J. C78 (2018), 566, arXiv:1712.05471

Dijets $\Delta\phi_{1,2}$

- opens up the ME phase space
- catches higher-order contributions
- \Rightarrow smaller parton shower correction needed



Summary and conclusions

- novel TMD evolution equation and new method (PB) to solve it
 - PB method provides full access to splitting kinematics $\Rightarrow k_T$
 - method consistency checked in integrated PDFs
 - PB solution at LO, NLO, NNLO
 - TMD determined, no extra parameters
- first TMD fit to data, including uncertainty band
 - TMD evolution implemented in xFitter
- application to LHC processes:
 - DY low p_T spectrum without extra parameters
 - dijet $\Delta\phi_{1,2}$: TMD enhances ME phase space
 - dijet $\Delta\phi_{1,2}$: PS correction drastically reduced \Rightarrow TMD catches higher order effects

Complementary slides

Where to find TMDs ? TMDlib and TMDplotter

- TMDlib proposed in 2014 as part of REF workshop and developed since
- combine and collect different ansaetze and approaches:

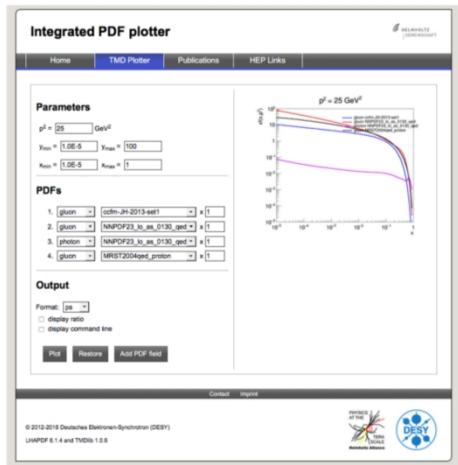
<http://tmd.hepforge.org/> and
<http://tmdplotter.desy.de>

- TMDlib: a library of parametrization of different TMDs and uPDFs (similar to LHApdf)

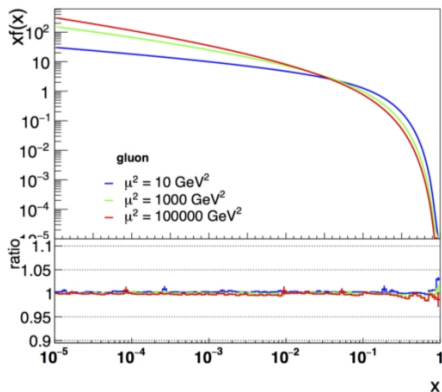
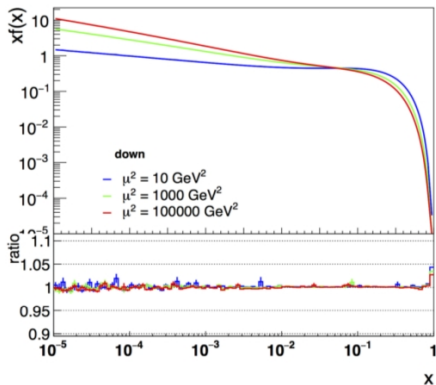
TMDlib and TMDplotter: library and plotting tools for transverse-momentum-dependent parton distributions, *F. Hautmann et al. arXiv 1408.3015, Eur. Phys. J., C 74(12):3220, 2014.*

- Also integrated pdfs (including photon pdf are available via LHAPDF)

- Feedback and comments from community is needed – just use it !



Validation of method with QCDnum at NLO



- Very good agreement with NLO - QCDnum over all x and μ^2
 - the same approach work also at NNLO !

MCEG: TMDs, parton shower

- basic elements are:
 - **Matrix Elements:**
 - on shell/off shell
 - **PDFs**
 - TMDs
 - **Parton Shower**
 - following TMDs for initial state !
- Proton remnant and hadronization handled by standard hadronization program, e.g. PYTHIA
- **Parton shower with TMDs follows exactly the evolution of the TMD**
 - no (!) free parameter in shower
 - resolvable branchings and calculation of k_T defined in TMD
 - no adjustment of kinematics during/after shower

